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Combined IMU Sensor Calibration and Bundle Adjustment with BINGO-F

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1 Introduction

Since aerial photos are used for map production – ever since photogrammetry has been a known technique – the estimation of the photo orientation has been a topic of research and development for mechanical engineers, photogrammetrists, mathematicians and software developers. New procedures and formulas have been invented and published all the time to facilitate this work. The wishful thinking to be able to do photogrammetry without this time consuming orientation work is slowly arriving – at least for selected applications – in a step by step fashion.

Actually, methods for direct measuring of orientation angles using inertial measuring units (IMU) of the two companies IGI (company 1 or C1) and Applanix (company 2 or C2) have been investigated in an OEEPE project. From both systems we can expect, that the orientation parameters can be estimated good enough for direct usage without bundle triangulation for the whole block, at least for applications with reduced precision requirements, e.g. for orthophoto production.

All participants of the project received the same data: Photo measurements of a calibration strip with C1= 62 and C2= 66 photos scale 1: 5 000, and a calibration block with C1= 86 and C2= 85 photos scale 1: 10 000. Furthermore the direct estimated projection center coordinates and orientation angles from the GPS/IMU systems for all photos of the block, C1= 284 and C2= 452 photos. From the calibration strips or blocks corrected orientation data have been predicted for all given photos.

The IMU misalignment angles are estimated in a combined adjustment. For the kinematic GPS observations shift and drift parameters have been applied. The mathematical model used will be described. All processing steps will be explained, documented and commented. Recognised problems will be discussed and recommendations for enhancements will be given. It will be summarised, that GPS/IMU application can help to avoid the time consuming bundle triangulation process for applications with lower precision requirements, e.g. for orthophoto production.

2 Used mathematical model

Because of physical reasons the IMU can never be mounted strictly parallel to the camera system. Therefore an equation system is required to describe the stabile relationship between the IMU and the camera to enable a simultaneous calibration in the bundle adjustment process. A mathematical rigorous approach has been developed. It has been applied for the OEEPE test fight in Norway.

The best results can always be achieved, if all available data are processed in a single computation step. This is the only way to take correlations as good as possible into account. As well the reliability will be increased and observations and results are better under control. Therefore the three rotation parameters of misalignment between the IMU and the camera have been introduced as additional unknowns in the bundle triangulation software BINGO-F. For this application a global shift has been estimated for all kinematic GPS data.

The following indices are introduced:

G Superior or ground coordinate system
I Instrument (i.e. IMU)
P Photo or photo coordinate system

The instrument **I** measures and records the orientation angles for all photos. The relation between measured rotational angles and the real photo orientations is given in equation (1):

$$\mathbf{R}_\mathbf{G}^\mathbf{I} = \mathbf{R}_\mathbf{G}^\mathbf{P} \cdot \mathbf{R}_\mathbf{P}^\mathbf{I} \qquad (1)$$

where

$\mathbf{R}_\mathbf{G}^\mathbf{I}$ Rotation from ground to instrument (observables of the IMU Φ, Ω, K).

$\mathbf{R}_\mathbf{G}^\mathbf{P}$ Rotation from ground to photo (orientation angles φ, ω κ)

$\mathbf{R}_\mathbf{P}^\mathbf{I}$ Rotation from photo to instrument . (constant)
(Misalignment calibration angles dφ, dω dκ or α, β, γ)

Rotational angles cannot be simply added together, if the basic (photo) system is already rotated. In case of terrestrial applications of photogrammetry there is another situation, if a camera is mounted on top of a theodolite and the orientation angles are estimated with the theodolite with vertical standing axes. There we have simply to add the differences between the theodolite and the camera. Those corrections can be understood as small corrections of the measuring pointer of the glass circle of the theodolite. But here in case of the IMU we have to multiply the rotational matrices.

Equation (1) describes the relation between the angles measured from the IMU and the photo orientation. For all three matrices the rotational sequence φ, ω κ is used. From this equation the observation equations (2) for Φ, Ω, K and their partial differential quotients have been established for iterative adjustment with BINGO-F.

$$\begin{aligned} \Phi + v\Phi &= f(\varphi, \omega, \kappa, d\varphi, d\omega, d\kappa) \\ \Omega + v\Omega &= f(\varphi, \omega, \kappa, d\varphi, d\omega, d\kappa) \\ K + vK &= f(\varphi, \omega, \kappa, d\varphi, d\omega, d\kappa) \end{aligned} \qquad (2)$$

The BINGO-F software includes of course all possibilities of full camera calibration, additional parameters, simultaneous estimation of a vector from the projection center to the antenna, corrections for gyro-mount readings, and much more. A complete description is found in the manual and partly as well in the literature [2].

3 Processing and results

In a first step all provided orientation angles have been converted from roll, pitch and yaw to φ, ω, κ for BINGO-F. The new angles have been corrected for meridian convergence. Therefore all further processing steps can be performed rigorous (with respect to the orientation angles) directly in the UTM coordinate system.

	Company 1				Company 2			
	Line_No.	*Shifts [mm] x*	*y*	*z*	*Line_No.*	*Shifts [mm] x*	*y*	*z*
1:5000 Shifts	*1087*	*-44*	*94*	*120*	*2004*	*-14*	*-68*	*353*
	1104	*-142*	*79*	*94*	*2022*	*27*	*47*	*340*
	1121	*-45*	*-2*	*59*	*2040*	*-79*	*12*	*331*
	1135	*-109*	*117*	*47*	*2055*	*24*	*10*	*329*
RMS GPS resid.:		*23*	*19*	*17*		*15*	*14*	*11*
Max GPS resid.:		*71*	*45*	*45*		*45*	*42*	*30*
	Line_No.	*Shifts [mm] x*	*y*	*z*	*Line_No.*	*Shifts [mm] x*	*y*	*z*
1:10000 Shifts	*1001*	*-121*	*19*	*31*	*2076*	*-68*	*124*	*490*
	1012	*-15*	*197*	*30*	*2087*	*112*	*-4*	*521*
	1024	*-160*	*-26*	*51*	*2098*	*-139*	*74*	*546*
	1035	*39*	*169*	*94*	*2109*	*85*	*-44*	*517*
	1046	*-183*	*140*	*162*	*2120*	*67*	*181*	*488*
	1061	*70*	*16*	*149*	*2135*	*-128*	*–154*	*396*
	1076	*-4*	*141*	*166*	*2150*	*73*	*-62*	*462*
RMS GPS resid.:		*15*	*17*	*16*		*99*	*75*	*44*
Max GPS resid.:		*45*	*45*	*51*		*331*	*177*	*133*

Tab. 1 GPS shift and drift parameters and GPS residuals

After the first adjustments systematic start-up or warm-up errors have been detected in the residuals of the first strips for both companies. For C1 in the calibration flight 1:5000 and for C2 in the calibration flight 1:10000. C1 provided later an enhanced data set with enhanced filtering with much better results.

The four adjustment processes are showing generally very good results, however, there are some differences. The results of C1 are looking generally slightly better that the results of C2 regarding homogeneity, maximum and RMS residuals of the IMU data and GPS data. An exception are the angle values of calibration flight 1:5000, here C2 has the smaller RMS values than C1.

Regarding the GPS data, we consider that for parts of the block the number of GPS satellites have been smaller for C1 than for C2. Especially for the calibration flight 1:10.000 for C2 with a very good satellite configuration, there are the highest discrepancies. However, we point out: All results are very good, because we are talking about a few decimeters only, as shown in Tab. 1.

The GPS shift and drift parameters are varying from strip to strip. This is an indication for incorrect fixing of phase ambiguity parameters. A new processing of the originally recorded GPS data should really be able to enhance the results, especially, if this would be done in a processing with GEONAP-K and BINGO-F, where GPS phase ambiguity estimation is integrated in a combined bundle adjustment and therefore much more reliable [2].

	[mgon]	phi	omega	kappa
Comp.2 / Cal. 1:10000	RMS residuals:	5.0	3.5	5.9
	Max residuals:	11.2	9.4	17.9
	rotat. angles:	-60.6	126.6	-197.1
	precision:	5.3	5.0	5.4
Comp.2 / Cal. 1:5000	RMS residuals:	14.6	11.1	11.2
	Max residuals:	40.4	27.0	20.5
	rotat. angles:	-59.1	130.6	-199.6
	precision:	5.6	5.2	5.7
Comp.1 / Cal. 1:10000	RMS residuals:	3.4	2.8	10.0
	Max residuals:	8.5	9.1	22.2
	rotat. angles:	-10.3	-99.6	66.6
	Precision:	5.1	4.8	5.1
Comp.1 / Cal. 1:5000	RMS residuals:	4.4	3.2	6.7
	Max residuals:	11.7	12.2	15.6
	rotat. angles:	-9.1	-104.0	66.9
	precision:	6.3	5.8	6.3

Tab.2 Results of IMU misalignment calibration
using ground control points

A surprise has been the differences in photo measurement precision: 4.0 µm for C1 and 5.8 µm for C2. These differences are related to the aircraft, the camera, the film development, the photo measurement device or the operator, but on no account to the GPS/IMU system. To avoid influences from these differences to the results of this test, the observation weights for each block have been individually adapted and optimised to the real measurement precision. Theoretically this will give the best accuracy. Empirical tests have confirmed this assumption.

These optimised weights have been used to estimate the adjusted misalignment angles of the IM. The measured IMU angles have been introduced with a high standard deviation (and a low weight) of 0.05 grads. The total redundancy in the variance component estimation confirms, that there is nearly no influence of these measurements to the adjustment results.

Tab. 2 gives an overview about all misalignment calibration results. For all four adjustments the RMS residuals *(RMS residuals)*, and the maximum residuals *(Max residuals)* of the measured IMU angles as well as the calibration angles *(rotat. angles)* and their standard deviations *(precision)* are presented. The misalignment angles have to be identical from both photo blocks. This fits in all cases very well within the given standard deviation.

In a further trial a processing without ground control points have been done. The results of the IMU misalignment calibration are identical (Tab.3). As well different trials with changes of some parameters resulted in the same angles.

	[mgon]	phi	omega	kappa
Comp.1 / Cal. 1:10000	RMS residuals:	3.4	2.8	10.1
	Max residuals:	12.5	9.5	21.8
	rot angles:	-10.4	-99.6	69.1
	precision:	5.1	4.8	5.1
Comp.1 / Cal. 1:5000	RMS residuals:	5.2	3.6	7.0
	Max residuals:	12.8	14.2	16.0
	rotat. angles:	-9.2	-104.1	67.8
	precision:	6.3	5.9	6.4

Tab.3 Results of IMU misalignment calibration
without ground control points

The residuals of all IMU angles are presented in Appendix A. Appendix B is an extract of the BINGO-F processing list file for all four adjustment processes.

The results of further considerations and processings are presented in [3]

3 Prediction of further orientation data

The results of the bundle triangulations from the calibration Blocks 1:5000 have been used to predict the orientation data of all remaining photos. For this purpose only a global shift was available for the whole block for the position, because there is no information about individual shifts of strips, which did not participate in the calibration process. For the orientation angles, all photo orientations have been multiplied with the calibration matrix.

I.e.: the results sent to the pilot center consists of :

- the original projection centers shifted by three global shift values for X, Y, Z,
- the given orientation angles corrected by a global rotation,
- the new values for the camera constant and principal point as well as some additional parameters.

The adjusted orientation parameters from the calibration block adjustments have not been used here.

4 Comparison with independent check points

IPI Hannover, the pilot center of this test, estimated the coordinates of independent check points from some photo measurements and the predicted orientation parameters. The results from all test participants are very good and better than RMS ~15 cm in planimetry and ~20 cm in height.

However, before we can conclude, that ALL estimated orientation data is good enough for ortho photo production or other purposes, the distribution and the maximum errors of all single rays compared to the independent check points should be known.

In [1] the pilot center concluded, that the Applanix (C2) results are better than the IGI (C1) results and in the range of some cm. There are several good reasons to plug a very big question mark upon this statement:

- The RMS precision values of adjusted point coordinates from bundle triangulations in photo scale 1:5000 are only about 3 cm in planimetry and 5 cm in height. For scale 1:10000 we have 5 and 10 cm.
- Looking to the variation of shift parameters in Tab. 1, precision values in the range of a few cm cannot be expected and are probably random numbers.
- We detected variations of the principal point position which will effect the ground coordinates probably more than 10 cm. Compare [2].
- The situation of the GPS satellites has been better during the C2 flight time than during the C1 flight time.

It cannot be said, that the computations of the pilot center have not been correct, however, it might be, that not all circumstances of the test have been acknowledged.

5 Conclusion

Both companies presented very good results. The differences in the results may be more influenced by the GPS coordinates than by the inertial measurement units (IMU). Therefore it is recommended to concentrate on the enhancement of GPS processing. The author presented in [1,2] better processing possibilities. These techniques are highly recommended for further investigations.

References

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Appendix A Residuals for all IMU angle measurements

Company 1 / Calibration 1:10000

Residuals [mgon]

photo	phi	omega	kappa
1001	6.4	6.9	7.8
1002	4.3	3.3	6.0
1003	8.5	4.9	4.4
1004	0.4	4.6	8.8
1005	1.6	1.0	7.5
1006	6.7	-1.1	9.5
1007	6.3	0.4	9.0
1008	5.9	2.7	10.1
1009	4.5	1.5	12.2
1010	7.9	-2.3	13.1
1011	5.8	0.5	9.6
1012	-3.2	3.1	14.2
1013	-0.1	-0.1	12.1
1014	0.9	1.7	15.6
1015	2.2	-1.7	16.6
1016	6.9	3.0	17.7
1017	1.3	-0.7	20.8
1018	-0.1	0.6	18.4
1019	0.2	2.6	18.2
1020	-2.9	0.3	17.0
1021	-4.6	1.8	18.6
1022	-1.9	0.7	17.4
1023	0.7	0.0	17.9
1024	-0.1	0.8	0.3
1025	1.7	-3.3	-4.2
1026	0.5	-3.3	-5.0
1027	-1.4	-3.8	-8.8
1028	4.0	-4.4	-7.2
1029	5.4	2.5	-7.1
1030	3.7	-1.7	-9.3
1031	4.7	-1.1	-11.3
1032	0.1	-0.9	-8.8
1033	0.1	3.5	-6.9
1034	2.6	1.9	-6.1
1035	-7.9	1.3	-5.8
1036	-4.6	0.3	-0.3
1037	-2.8	-1.5	1.1
1038	-4.7	0.8	-0.5
1039	5.4	0.0	-1.2
1040	0.3	0.9	3.2
1041	4.5	2.8	0.4
1042	6.0	3.0	1.0
1043	-0.2	-2.8	7.1
1044	1.4	-2.1	5.6
1045	2.9	-4.4	8.3

Residuals [mgon]

photo	phi	omega	kappa
1046	-5.5	0.1	-3.6
1047	0.9	3.0	-3.2
1048	-0.8	2.6	-1.4
1049	2.8	-6.1	-4.0
1050	-0.3	-5.9	-1.1
1051	0.5	-1.0	-0.9
1052	-1.8	-1.3	-2.2
1053	-0.9	-4.1	0.6
1054	-1.8	-1.1	-0.8
1055	-0.8	-0.1	-0.5
1056	-0.8	-5.8	1.7
1057	2.2	-0.1	0.0
1058	-0.6	0.6	5.4
1059	-2.1	2.5	3.1
1060	-0.2	-0.8	3.7
1061	-4.8	1.1	-4.5
1062	0.3	-9.1	-5.4
1063	-0.5	-3.1	-5.9
1064	-0.8	0.0	-4.9
1065	0.6	0.2	-5.0
1066	1.0	-4.1	-4.2
1067	0.6	-0.5	-3.2
1068	3.2	-1.4	-3.9
1069	0.6	-0.5	-5.8
1070	-1.7	1.7	-4.9
1071	2.1	0.4	-3.9
1072	0.1	-3.5	-5.9
1073	2.3	3.4	-4.0
1074	-2.9	1.3	-5.5
1075	-0.7	0.0	-7.2
1076	1.5	-5.4	-22.2
1077	-0.4	-2.0	-20.6
1078	1.1	-0.8	-19.3
1079	2.0	-1.8	-15.3
1080	4.7	-0.9	-16.5
1081	3.0	-4.2	-15.6
1082	4.0	-3.5	-15.4
1083	-1.9	-4.2	-13.8
1084	6.0	-0.7	-11.8
1085	1.3	-1.7	-12.3
1086	0.5	1.3	-6.7

	phi	omega	kappa
RMS resid.:	3.3	2.8	10.0
rot angles:	-10.3	-99.6	66.6
Max resid.:	8.5	9.1	22.2
precision:	5.1	4.8	5.1

Company 1 / Calibration 1:5000

Residuals [mgon]

photo	phi	omega	kappa
1087	-2.2	0.6	0.3
1088	1.7	1.6	2.1
1089	1.8	0.0	2.6
1090	3.4	0.0	3.8
1091	4.3	-1.4	4.6
1092	8.8	-0.7	4.7
1093	4.0	-3.8	4.1
1094	1.7	-2.9	3.6
1095	0.5	-2.4	2.1
1096	-5.7	-0.6	4.3
1097	-0.2	0.7	7.0
1098	-1.5	1.3	4.7
1099	3.4	0.4	5.2
1100	-1.3	-1.4	4.7
1101	-1.6	-2.1	7.1
1102	-4.7	-2.6	5.1
1103	-6.8	-1.3	6.2
1104	2.4	-1.7	0.4
1105	5.4	-1.5	-1.2
1106	3.9	-0.4	4.8
1107	3.9	0.0	1.2
1108	1.7	-0.6	1.8
1109	-3.4	2.6	0.9
1110	0.5	1.8	4.7
1111	7.2	1.9	-1.1
1112	2.5	3.7	3.4
1113	4.9	2.5	2.1
1114	4.3	1.3	2.5
1115	4.4	-0.9	2.9
1116	5.6	4.4	3.4
1117	4.4	1.5	4.5
1118	3.4	1.3	5.1
1119	2.6	3.6	2.1
1120	-1.1	6.0	3.5

Residuals [mgon]

photo	phi	omega	kappa
1121	2.7	1.3	-14.7
1122	0.1	-0.3	-9.0
1123	2.1	-6.0	-10.9
1124	5.0	1.0	-11.3
1125	1.7	-2.6	-10.3
1126	4.5	-4.6	-12.2
1127	5.0	-3.4	-10.2
1128	1.4	-0.1	-10.4
1129	2.6	-1.1	-10.6
1130	5.3	-3.4	-10.5
1131	8.9	-0.9	-14.4
1132	3.5	1.0	-10.3
1133	4.5	-9.1	-14.4
1134	9.0	-12.2	-15.6
1135	6.2	-5.2	0.5
1136	7.2	-4.3	2.5
1137	8.3	-4.1	1.1
1138	11.7	1.2	0.1
1139	1.4	1.4	3.3
1140	4.1	1.6	3.4
1141	2.2	-1.4	2.8
1142	5.8	3.7	2.8
1143	1.1	5.6	6.5
1144	2.5	-0.2	3.4
1145	0.1	1.0	9.2
1146	1.1	-3.4	6.7
1147	-0.6	-4.7	5.7
1148	-1.6	-0.1	3.7

RMS resid.:	4.4	3.2	6.7
Max resid.:	11.7	12.2	15.6
rot angles:	-9.1	-104.0	66.9
precision:	6.3	5.8	6.3

Company 2 / Calibration 1:10000

Residuals [mgon]

photo	phi	omega	kappa
2076	3.6	5.5	-3.2
2077	4.2	0.8	-5.3
2078	2.7	4.8	-10.2
2079	2.9	0.1	-9.8
2080	3.1	2.0	-12.6
2081	0.7	4.3	-11.1
2082	6.0	1.4	-9.2
2083	3.6	3.1	-8.3
2084	0.2	-2.8	-7.6
2085	-2.1	1.9	-10.6
2086	-1.0	6.0	-17.9
2087	-8.4	-8.6	4.4
2088	-5.7	-0.1	-3.9
2089	-2.3	1.0	-4.7
2090	-4.1	0.7	-3.5
2091	1.6	-3.9	-2.5
2092	2.8	1.7	-5.1
2093	5.6	1.6	-2.6
2094	4.8	0.8	-1.8
2095	5.9	1.0	-3.1
2096	9.7	1.6	-7.7
2097	11.2	2.4	-10.2
2098	4.5	-0.1	-1.4
2099	5.7	-4.9	4.1
2100	7.2	-2.0	4.0
2101	4.7	-1.9	4.6
2102	5.2	-0.9	2.5
2103	2.8	-2.1	3.5
2104	6.8	-3.2	3.8
2105	2.9	-3.1	3.7
2106	0.7	-1.8	5.2
2107	-0.8	-3.6	3.3
2108	-2.7	-3.2	2.9
2109	-6.2	-3.3	15.2
2110	-5.7	-0.8	9.1
2111	-0.3	-2.3	4.9
2112	-1.4	-0.5	7.6
2113	1.8	-2.7	8.5
2114	4.7	1.1	4.3
2115	5.0	0.1	5.7
2116	5.7	-2.0	9.3
2117	5.3	2.5	7.8
2118	7.6	-0.9	7.3
2119	10.7	-4.8	0.6

Residuals [mgon]

photo	phi	omega	kappa
2120	1.8	7.9	-1.2
2121	5.3	8.3	-2.9
2122	4.3	4.5	0.5
2123	4.6	4.6	-0.3
2124	4.8	2.7	-1.3
2125	5.8	3.9	-1.1
2126	6.3	4.3	-3.1
2127	6.1	1.4	-6.1
2128	6.1	-0.1	-4.8
2129	3.4	-0.2	-5.0
2130	2.8	-0.9	-2.2
2131	2.9	-1.3	1.0
2132	7.6	-3.5	0.6
2133	4.0	-4.2	-2.1
2134	3.5	-6.5	-2.8
2135	7.2	-4.8	2.1
2136	5.2	-8.1	-1.9
2137	5.3	-3.4	1.8
2138	7.0	-3.7	-0.4
2139	6.3	-1.8	3.9
2140	5.9	1.7	-0.2
2141	6.6	-0.6	0.5
2142	1.4	1.9	-0.2
2143	5.9	-0.6	1.5
2144	2.8	-1.6	-1.3
2145	5.2	0.7	1.6
2146	3.1	1.5	0.7
2147	3.7	6.7	3.1
2148	2.3	7.9	5.9
2149	-0.1	9.4	1.9
2150	-4.0	-2.5	12.1
2151	-2.2	3.2	7.0
2152	0.4	-1.8	6.9
2153	2.3	0.0	6.2
2154	2.7	-0.5	6.4
2155	5.8	0.2	4.8
2156	1.1	-1.4	4.4
2157	3.9	0.2	3.1
2158	4.8	1.2	-0.7
2159	7.5	1.4	-1.2
2160	8.5	0.6	-7.1

RMS resid.:	5.0	3.5	5.9
Max resid.:	11.2	9.4	17.9
rot angles:	-60.6	126.6	-197.1
precision:	5.3	5.0	5.4

Company 2 / Calibration 1:5000

Residuals [mgon]

photo	phi	omega	kappa
2004	-7.8	-5.6	-20.5
2005	-9.7	2.6	-17.5
2006	-0.3	-5.8	-14.9
2007	-0.6	-6.4	-15.3
2008	-5.8	1.5	-14.0
2009	-2.0	0.0	-16.4
2010	4.0	-5.3	-16.0
2011	9.1	3.1	-14.0
2012	10.3	4.2	-15.7
2013	15.0	5.2	-15.6
2014	16.8	6.9	-14.5
2015	16.1	7.0	-12.5
2016	21.0	9.5	-11.6
2017	26.7	12.8	-9.6
2018	28.3	9.3	-10.6
2019	35.4	11.8	-11.1
2020	34.9	10.4	-11.0
2021	40.4	21.6	-10.9
2022	33.1	16.5	18.6
2023	36.9	14.8	19.8
2024	29.9	10.7	15.7
2025	28.9	12.5	20.5
2026	22.9	11.9	17.8
2027	21.1	8.6	16.9
2028	14.8	5.9	16.9
2029	13.7	2.8	17.3
2030	10.6	3.1	14.6
2031	10.8	5.5	14.8
2032	6.5	-0.8	16.2
2033	5.3	0.8	11.7
2034	7.2	6.7	10.5
2035	-4.0	-0.4	12.2
2036	-2.6	-1.5	11.1
2037	-0.9	-2.2	9.5
2038	-7.1	-3.3	7.3
2039	-10.6	-2.2	4.4
2040	6.8	-8.1	0.3

Residuals [mgon]

photo	phi	omega	kappa
2041	3.8	-2.5	1.3
2042	11.6	-4.0	6.6
2043	1.6	-3.7	5.2
2044	2.2	-0.9	4.1
2045	3.4	5.6	3.0
2046	2.8	6.3	5.3
2047	0.6	6.2	3.6
2048	-0.6	11.3	3.6
2049	-6.0	14.1	5.5
2050	-7.0	17.1	6.3
2051	1.0	19.7	3.1
2052	-3.8	23.7	2.6
2053	-6.5	27.0	3.5
2054	-0.4	26.8	1.2
2055	-8.0	25.2	0.5
2056	-2.4	25.6	0.6
2057	-5.5	15.8	-2.2
2058	-2.9	17.6	-2.1
2059	-4.3	14.1	-1.3
2060	-4.5	8.3	-0.7
2061	-2.1	6.4	-3.6
2062	-2.2	8.1	-1.9
2063	1.6	4.8	-5.3
2064	3.1	6.2	-4.3
2065	3.6	-1.8	-5.8
2066	7.2	-0.9	-6.9
2067	2.9	-2.7	-6.0
2068	5.4	-5.5	-11.0
2069	7.8	-6.7	-8.9

RMS resid.:	14.6	11.1	11.2
Max resid.:	40.4	27.0	20.5
rot angles:	-59.1	130.6	-199.6
precision:	5.6	5.2	5.7

Appendix B Extract from the BINGO List File (Example for one adjustment)

```
                              BINGO-F  -  VERS. 4.0 / 10.00d
===============================================================================
                                Company 1 / Cali 10
-------------------------------------------------------------------------------
                                       :
                                       :
                                       :
  Summary of image data:
  ----------------------
  No. of points                :     319
  No. of photos                :      86
  No. of cameras               :       1
  Max. measurements per point  :      22
  Max. photo index difference  :      63

                                       :
                                       :
                                       :
                                                SIGMA 0 =       3.79 (1/1000)
===============================================================================

          Camera data
-------------------------------------------------------------------------------

 Camera no. 1

     Diff. angle of rotation delta :   -0.0103   -0.0996    0.0666
     +-S                   (1/1000) :       5.1       4.8       5.1

     Additional parameters         :   Format factor =      1.000000

            7         8        17

        0.0165   -0.0055    0.0035


     Mean radial symmetric lens distortion from additional parameters  (1/1000)

     Distortion values; First value for R =  10.0  (= Step width)

         1.8     3.4    4.6    5.5    6.1    6.5    6.4    6.1    5.3    4.3
         2.8     1.0   -1.1   -3.7   -6.7  -10.1


 Correlation between add. parameters in %

   7   100

   8   -63 100

  17    -6    0 100

         7    8   17


 Par.no  Parameter value   Standard dev.   Value/Stand.dev   Total correlation
               (1/1000)          (1/1000)

     7           16.5             0.6            29.4             0.41
     8           -5.5             1.0            -5.6             0.41
    17            3.5             0.2            14.8             0.01
```

GPS shift and drift parameters

File	Line_No.	Para.Name	Shift	Drift	+-S	Photos
1	1001	s_X	-0.196		0.030	11
1	1001	s_Y	0.114		0.022	11
1	1001	s_Z	-0.029		0.017	11
1	1001	d_X		-0.029	0.064	11
1	1001	d_Y		-0.015	0.044	11
1	1001	d_Z		-0.048	0.038	11
1	1012	s_X	0.079		0.024	12
1	1012	s_Y	0.100		0.019	12
1	1012	s_Z	-0.034		0.014	12
1	1012	d_X		0.051	0.052	12
1	1012	d_Y		-0.043	0.041	12
1	1012	d_Z		0.037	0.035	12
1	1024	s_X	-0.234		0.024	11
1	1024	s_Y	0.096		0.019	11
1	1024	s_Z	-0.013		0.015	11
1	1024	d_X		-0.111	0.048	11
1	1024	d_Y		0.066	0.040	11
1	1024	d_Z		0.045	0.033	11
1	1035	s_X	0.156		0.031	11
1	1035	s_Y	0.054		0.024	11
1	1035	s_Z	0.030		0.019	11
1	1035	d_X		-0.025	0.063	11
1	1035	d_Y		0.053	0.051	11
1	1035	d_Z		-0.032	0.041	11
1	1046	s_X	-0.061		0.021	15
1	1046	s_Y	0.230		0.020	15
1	1046	s_Z	0.099		0.014	15
1	1046	d_X		-0.075	0.055	15
1	1046	d_Y		0.037	0.061	15
1	1046	d_Z		0.120	0.051	15
1	1061	s_X	-0.036		0.020	15
1	1061	s_Y	-0.066		0.020	15
1	1061	s_Z	0.086		0.014	15
1	1061	d_X		-0.019	0.063	15
1	1061	d_Y		0.055	0.072	15
1	1061	d_Z		-0.048	0.059	15
1	1076	s_X	0.097		0.023	11
1	1076	s_Y	0.039		0.019	11
1	1076	s_Z	0.102		0.015	11
1	1076	d_X		0.018	0.052	11
1	1076	d_Y		-0.014	0.044	11
1	1076	d_Z		0.018	0.036	11

```
         Exterior orientation data

Type   Photo        X             Y            Z          Phi       Omega      Kappa
          +- S (1/1000)
--------------------------------------------------------------------------------

 A       1001    611157.126  6571321.046   1608.147   -1.0285     0.7827 -132.6471
           +-           56.           48.        41.       1.9        1.9        1.9
                                        :
                                        :
                                        :
 A       1086    612848.529  6570622.381   1614.551    0.6820    -0.2854   77.0392
           +-           49.           43.        36.       1.7        1.6        1.6

     Mean photo scale: 10.0

     RMS precision values of photo orientations from Qxx matrix: (1/1000)

                        41.          36.        26.       1.4        1.3        1.0

     Poorest precision values of photo orientations from Qxx matrix: (1/1000)

                        74.          59.        49.       2.5        2.1        2.3



     Listing of object point coordinates suppressed.
                                                                   +- S X      S Y      S Z
                                                                       ( 1 / 1 0 0 0 )

     RMS precision values of object points:                             39.      35.      72.
     Poorest precision values of object points:                        117.     160.     190.

     RMS precision values of control points:                             9.       9.       9.
     (Computed from Qxx matrix)

                                        :
                                        :
                                        :

          Residuals of image coordinates    list limit =  3.0 * sigma

        Point Photo     Vx'      Vy'     Rx'% Ry'%    Wx'   Wy'     Nabla x'  y'
                      ( 1 / 1 0 0 0 )                               ( 1 / 1 0 0 0 )
--------------------------------------------------------------------------------

**         2136
               1012     -3.1      9.9     33   57    -1.3   3.3        9.1   -17.1
               1011     -0.4      3.7     66   62    -0.1   1.2
               1013      0.4     -7.1     69   66     0.1  -2.2
               1010      0.2      0.8     59   60     0.1   0.3
               1014      2.1      1.2     59   66     0.7   0.4
                                        :
                                        :
                                        :
**          463
               1005      0.4     -7.8      0   41 unreal.  -3.0   unreal.    18.6
               1006     -0.4      7.9      0   43    -3.0   3.0     331.5   -18.3

Number of skipped photo measurements:    10
```

```
Frequency of photo measurement residuals  N(0,1) :
                  for x                                   for y

                    *
                    *
                    *
                    *                                      * *
                   ***                                     ***
                  ****                                    *****
                 *******                                 *******
                ********                                *********
               ***********                             ***********
         *************************              *************************  *
 <------------------+------------------> <------------------+------------------>
  - 4   3   2   1   0   1   2   3   4 +   - 4   3   2   1   0   1   2   3   4 +

                                       :
                                       :
                                       :
```

RMS control point residuals:	4.	4.	3.	(1/1000)
Maximum control point residuals:	9.	9.	6.	(1/1000)
RMS GPS residuals:	14.	16.	13.	(1/1000)
Maximum GPS residuals:	37.	44.	47.	(1/1000)

(Computed from real residuals)

A posteriori variance-component estimation

Test value = s(a posteriori) / s(a priori)

Group		Test Value	No. of Obs.	Redundancy
Image coordinates	:	0.98	4756	3443.19
Coordinates of control points	:	1.02	39	5.16
Control points only in X	:	0.98	13	2.16
Control points only in Y	:	0.92	13	2.18
Control points only in Z	:	1.35	13	0.82
Image station information	:	0.13	258	254.83
Exterior orientations incl. GPS	:	0.87	258	86.82
Sum of all observations	:	0.95	5311	